COMMENTS

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Shell closure at $N=164$: Spherical or deformed?

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Brenner *et al.* [1] recently reported the apparent evidence for a spherical shell at $N=164$. Some arguments are given which may make it necessary to reconsider this conclusion. [S0556-2813(97)02702-7]

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Brenner *et al.* [1] compared the systematics of the empirical ratio (ϵ/Δ), with the calculated P factor for nuclei with atomic number $A>200$. Here, (ϵ) is the nuclear quadrupole deformation, (Δ) is the pairing gap-parameter, and $P=(N_pN_n)/(N_p+N_n)$, where N_p and N_n are the valence protons and neutron numbers respectively. The P factor can be viewed as proportional to the ratio of the valence p-n interactions to the pairing interaction. Thus, it is logical to expect P to be correlated with an empirical measure of the ratio (ϵ/Δ).

Actually, in Ref. [2] a remarkable correlation was observed between the empirical values of (ϵ/Δ) and the P factor for all even-even nuclei for $Z=42$–98. Nevertheless, these authors observed [1] that the P-factor values in the actinide region, cannot reproduce quite well the experimental (ϵ/Δ) systematics within the framework of traditional shell closures, $Z=82$, 126 and $N=126$, 184. They found that the P factor systematics reproduces more accurately the features of the experimental ratio if a shell closure is presumed at $N=164$.

This result is very interesting because studying the decay properties of several new isotopes with atomic numbers $Z=106$, 108, 110, and neutron numbers near 162, it has been established [3–5] that a huge increase of their stability occurs as compared to the predictions of models which do not take into account the presence of a new shell [6,7].

Furthermore, the α-particle energy measured [5,8] for the $N=163$ isotope $^{273}110$, $(E_\alpha>11$ MeV) provides direct and convincing evidence that a neutron shell closure indeed exists and is located at $N=162$ and not at a higher value of N. The E_α value for $^{273}110$ would have been about 1 MeV lower if the shell closure had occurred at $N>162$.

The existence of a new shell closure with $N=162$ had been predicted in several theoretical papers [10–12]. But, in contrast with the conclusion of Brenner *et al.* [1], the predicted new shell ought to be deformed, not spherical.

Experiments like those described in Refs. [2–4,8,9] do not allow us to determine if the new shell with $N=162$ corresponds to a spherical shell or to a deformed one. However, the coincidence between the experimental values of such quantities such as α-decay energy, half life, etc., with the theoretical predictions (see for example [11]), gives an indirect indication of the possible deformations of these nuclei.

In [1], the authors did not discuss why they called the shell with $N=164$ spherical. Apparently, the only reason that they obtained the value $N=164$ from Ref. [13], where an improved parameter fit for the deformed nuclear potential is made. But, looking at Fig. 5 in [13], one will observe that the possible spherical neutron shells with $N=126$, 136, 170, 184, and 196, were emphasized, but not any with $N=164$. Certainly, there is a shell emphasized with the number 164, but it corresponds to a proton shell $(Z=164)$.

In addition, an important property of these heavy nuclei with N near 164 is their fission ability. The fission process is characterized by a rather sharply defined threshold energy, referred to as the fission barrier. In [13], it is stressed that the detailed dependence of the barrier on N and Z appears not to be reproduced by their model (see page 625 in [13]).

On the other hand, Brenner *et al.* [1] noted that a deformed subshell gap at $N=152$ could influence the fine structure observed around the proton number $Z=90$ [see Fig. 2 in [1]]. Because the influence of the deformed shell with $N=162$ may be stronger than the one with $N=152$ [11], we would expect that the P-factor values be influenced by that shell. Moreover, the differences in the P-factor values calculated for a supposed shell with $N=164$, are not dramatically different from those calculated by considering a shell with $N=162$.

In any case, it is not clear how to demonstrate whether or not a certain shell is spherical on the basis only of the nucleon numbers in the shell closure. From the results obtained by Heiss *et al.* [14,15], it follows that the same magic numbers can be reproduced by a purely quadrupole deformed Hamiltonian, or with a combination of the same Hamiltonian with higher multipoles.

So, it is my opinion that the results of Brenner *et al.* [1] could be interpreted as a possible confirmation of the presence of a deformed shell with $N=162$.

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